ANALYTICAL STUDY ON SIMPLY SUPPORTED RECTANGULAR SLAB WITH OPENING

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UNIVERSITI MALAYSIA SARAWAK

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CIVIL ENGINEERING PROGRAM

FACULTY OF ENGINEERING

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Universiti Malaysia Sarawak
Kota Samarahan

fk

BORANG PENYERAHAN TESIS

Judul: Analytical Study On Simly Supported Rectangular Slab With Opening.

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Tarikh: 19/3/03

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE BACHELOR OF ENGINEERING (CIVIL ENGINEERING) 2003

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FACULTY OF ENGINEERING
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APPROVAL SHEET

This Final Year Project Report entitled "ANALYTICAL STUDY ON SIMPLY SUPPORTED RECTANGULAR SLAB WITH OPENING" prepared and submitted by DANIEL LEE KIM TEE in partial fulfillment of the requirement for the Bachelor of Engineering (Civil) is hereby accepted.

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ABSTRACT

Reinforced concrete slabs are used to provide flat, useful surfaces and commonly appear in floors, roofs and deck of bridge. Many slabs exist in more complex cases that do not meet the constraints of elasticity-based method as a result of shape, support conditions or the presence of openings. Then elasticity-based method is not applicable in the analysis of slab with opening. But limit analysis or plastic analysis is an alternative to treat this problem. So yield line analysis that based on limit analysis has been introduced to analyze the slab with opening.

This project involves analytical study on simply supported rectangular slabs with opening by using yield line analysis based on limit analysis. Yield line analysis is utilized to determine the internal and external work done on slab. By equating internal and external work done, relation between applied loading per unit area and ultimate resisting load can be obtained. The procedures stated above applicable to both cases either for ordinary rectangular slab and rectangular slab with opening. Then the effect on load carrying capacity of slab at different dimensions with varying size of opening can be observed.

For the purpose of time saving and accuracy, MathCAD program has been utilized in this project to assist in obtaining values of load carrying capacity at various ratios of slab and opening dimension.

ABSTRAK

Papan konkrit yang diperkuatkan digunakan untuk membekalkan permukaan yang rata dan berguna. Ia biasa digunakan pada lantai, bumbung dan lantai jambatan.. Kebanyakan papan konkrit muncul dalam kes yang lebih komples dan tidak dapat memenuhi syarat-syarat penggunaan teori elastik atas sebab bentuk, keadaan penyokong atau kehadiran kekosongan pada papan. Jadi analisa elastik tidak sesuai dipergunakan pada papan konkrit yang mempunyai kekosongan. Akan tetapi analisa keplastikan merupakan satu alternatif untuk mengatasi masalah ini. Jadi teori garis lentur yang berasaskan analisa keplastikan telah dipergunakan untuk menganalisa papan konkrit yang kehadiran kekosongan.

Projek ini merangkumi pembelajaran secara analisa mengenai papan konkrit yang disokong secara ringkas dengan kehadiran kekosongan. Teori garis lentur telah digunakan untuk memperolehi kerja dalaman dan luaran pada papan konkrit. Dengan mempersamakan kerja dalaman dan luaran, satu perhubungan antara beban ditanggung pada satu unit luasan dan beban maksimum yang dapat dirintang dapat diperolehi. Prosidul yang dicatatkan seperti di atas dapat dipergunakan pada papan konkrit biasa dan papan konkrit yang mempunyai kekosongan. Jadi kesan pada keupayaan menanggung beban papan konkrit pada dimensi papan konkrit dan kekosongan yang berlainan dapat diperhatikan.

Untuk menjimatkan masa dan memperolehi hasil yang tepat, program MathCAD telah digunakan untuk menolong mendapatkan nilai-nilai keupayaan menanggung beban pada dimensi papan konkrit dan kekosongan yang berlainan.

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A_s = Area of steel

$A_{s.req.}$ = Area of steel required

$A_{s.prov.}$ = Area of steel provided

b = Width of slab

b' = Distance of an opening from the edge of the slab

d = Effective depth

d' = Effective depth for compression reinforcement

f_{cu} = Concrete characteristic strength

f_y = Steel reinforcement characteristic strength

f_s = Estimated design service stress

G_k = Dead load

K = Design constant

K' = Reference for design constant

ℓ = Length of the yield line

L_x = Length on the shorter span of the slab

L_y = Length on the longer span of the slab

L_{ox} = Length of an opening in shorter span direction

L_{oy} = Length of an opening in longer span direction

m = Moment of resistant

m' = Negative moment

m_N = Moment on neutral axis

m_x = Moment on short span

m_y	=	Moment on long span
m_T	=	Moment on T axis
Mult.	=	Ultimate moment
Q_k	=	Imposed load
v	=	Nominal design shear stress
v_c	=	Design ultimate shear stress
V	=	Maximum shear at support
w	=	Ultimate loading per unit area
w_{ult}	=	Ultimate loading
Z	=	Lever arm
θ	=	Rotation in the yield line
δ	=	Deflection on slab
α_{sx}	=	Moment coefficient in short span
α_{sy}	=	Moment coefficient in long span
β_b	=	Ratio of moments after and before redistribution
γ_m	=	Partial safety factor

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Reinforced concrete slabs are among the most common structural elements. In reinforced concrete construction slabs are used to provide flat, useful surfaces. A reinforced concrete slab is a broad, flat plate, usually horizontal, with top and bottom surfaces parallel or nearly so. They can be used in floors, roofs and walls of buildings and as well as the deck of bridges. Generally slabs may be divided into two categories: beamless slabs and slabs supported on beams located on all sides of each panel or known as simply supported slab. Beamless slabs are described by generic terms flat plates and flat slabs.

1.2 SIMPLY SUPPORTED RECTANGULAR SLAB

A rectangular slab transfers loadings to its adjacent supports. The under loading simply supported rectangular slab will have one of the structural actions in major: one-way spanning and two-way spanning.

One-way spanning slab can be distinguished from two-way spanning slab by its cylindrical bending shape when loaded. Consider a rectangular slab that shown in Fig 1.1 that is simply supported along its two opposite long edges and free of any support along the two opposite short edges. If a uniformly distributed load is applied to the surface of the slab, the deflected shape will be as shown by solid lines. Curvatures, and consequently bending moments, are the same in all strips s spanning in the short direction between supported edges, whereas there is no curvature, hence no bending moment, in the long strips l parallel to the supported edges.

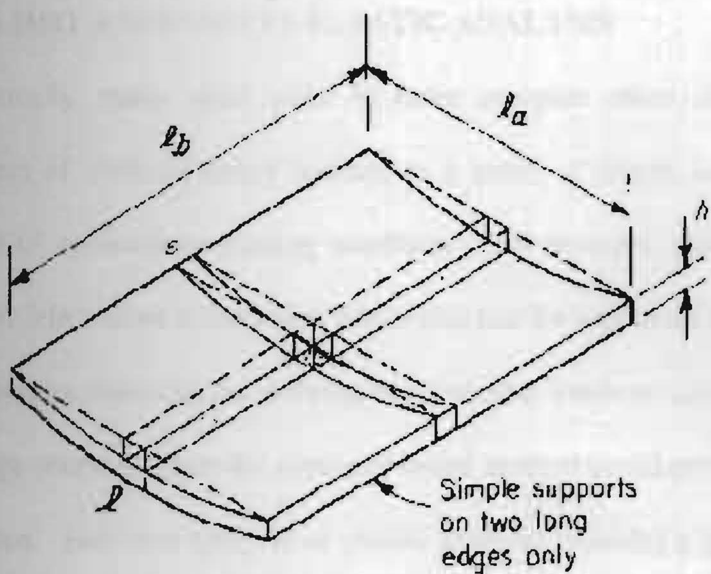

Fig 1.1 Deflected shape of uniformly loaded one-way slab.

Two-way spanning slab that shown in Fig 1.2 will bend into a dished surface when loaded. This means that at any point the slab is curved in both principal directions, and since bending moments are proportional to curvatures, moments also exist in both directions. It is convenient to think the two-way slab consisting of two sets of parallel strips, in each of the two directions, intersecting each other. Evidently, part of the load is carried by one set and transmitted to one pair of edge supports, and the remainder by the other.

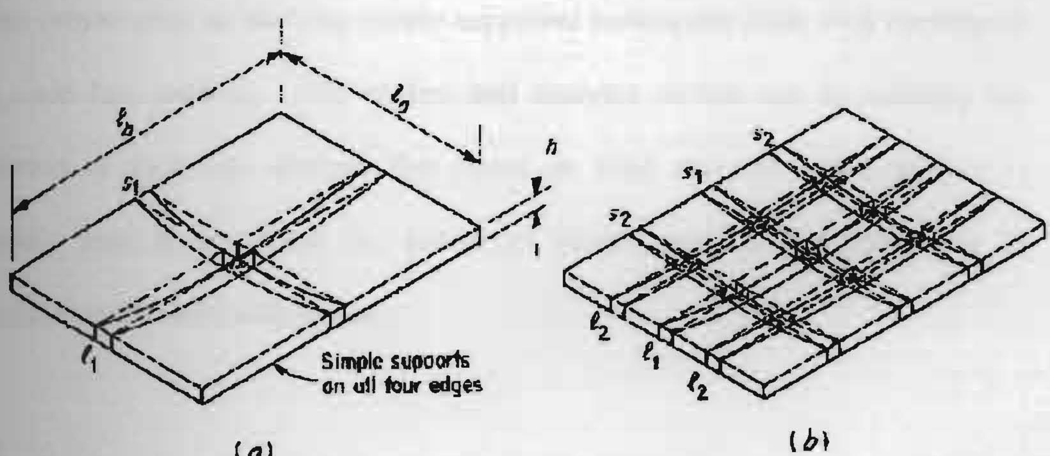

Fig 1.2 Two-way slab on simple edge supports: (a) bending of center strip of slab; (b) grid model of slab.

1.3 LIMIT ANALYSIS VS ELASTIC ANALYSIS

Practically, many slabs exist in more complex cases that do not meet the constraints of elasticity-based method as a result of shape, support conditions, the presence of openings or loading condition. For example, slab panel shape can be triangular, circular or trapezoidal. Slabs also can be supported on two or three edges only. Besides slabs can be carrying concentrated loads or uniform loads. Slabs can have large opening. Then the elasticity-based method could not be utilized due to the restrictions. But limit analysis or plastic analysis provides a powerful and versatile tool for treating this problem.

Slabs typically have tensile steel ratios much lower than the balanced fracture value and consequently have large rotation capacity. Therefore it can be safely assumed that the necessary ductility is present. Yield line theory is one of the practical methods that have been developed and being utilized in this project for the plastic analysis of slabs.

1.4 OBJECTIVE

This project aims at studying simply supported rectangular slabs with opening by using yield line analysis. The studies and analysis carried out by utilizing the application of yield line analysis that based on limit analysis. Limit analysis is practically used to overcome the barrier of elastic analysis that restricting its application on the slabs with opening.

CHAPTER 2

LITERATURE REVIEW

2.1 GENERAL

Most concrete slabs are designed for moments found by methods that are based upon elastic theory. The actual proportioning of members is done by strength methods, with the recognition that inelastic section and member response would result upon overloading. Factored loads are used in the elastic analysis to find moments in slab, for example, after which the critical slab section is designed with the knowledge that the steel would be well into the yield range and the concrete stress distribution very nonlinear before final collapse. Although it can be shown to be both safe and conservative, this is clear that there is existence of an inconsistent approach or contradiction to the total analysis-design process.

Thus limit or plastic analysis of reinforced concrete was introduced. Limit analysis does not only eliminates the inconsistency of combining elastic analysis with inelastic design but also accounts for the reserve strength characteristic of most reinforced concrete structures and permits, within limits, an arbitrary readjustment of moments found by elastic analysis to arrive at design moment that permit more practical reinforcing arrangement.

Kong and Evans (1975) stressed that there are three important aims in reinforced concrete design. They have suggested that;

1. The structure must be safe, for society demands security in the structure it inhabits.
2. The structure must fulfill its intended purpose during its intended life span.

3. The structure must be economical with regards to the first cost and to maintenance costs; indeed, most decisions are implicitly or explicitly economic decision.

2.2 ANALYSIS AND DESIGN OF A RECTANGULAR SLAB

2.2.1 Slab Action

This section discusses on floor slabs supported at four edges in two-way spanning action as shown in Fig. 2.1. Since it is a rectangular slab, more than one-half of the load will be carried in the stiffer or shorter direction and less in the longer direction.

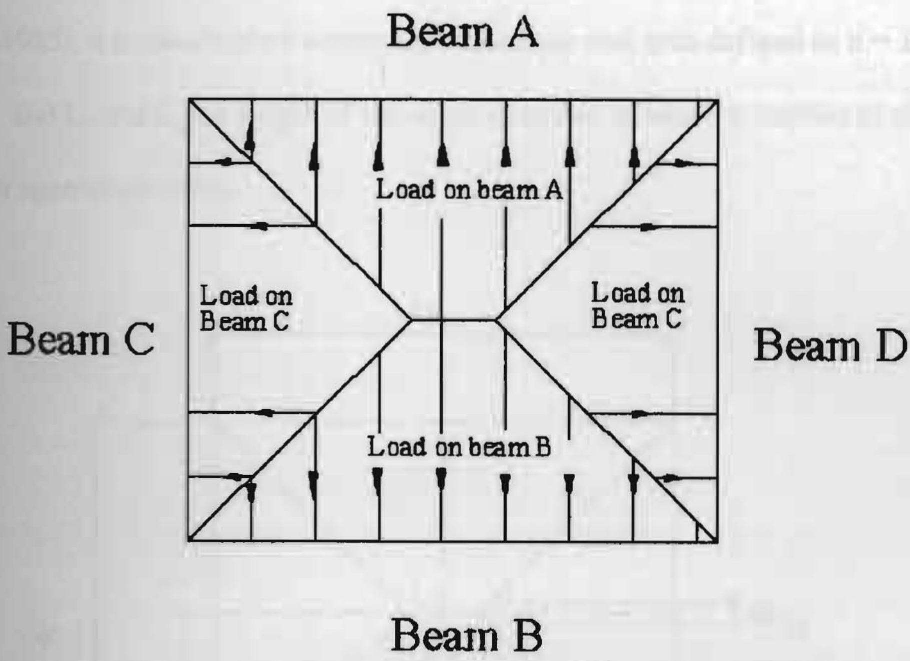

Fig 2.1 Distribution of two-way action of rectangular slab that supported by beams.

2.2.2 Analysis and Design of Two-way Spanning Slab

Referring to Fig 2.1, taking into account that the rectangular slab is simply supported at each edge and discontinuous at the edges. The design support does not have adequate provision either to resist torsion at corners or to prevent the corners from lifting are based on Clause 3.5.3.3 of Part 1 of BS8110 (1985). Based on the clause, it is defined that the equations for calculation of ultimate moment m_{sx} and m_{xy} are given as;

$$m_{sx} = \alpha_{sx}\, n\, L_x^2 \tag{2.1}$$

$$m_{sy} = \alpha_{sy}\, n\, L_y^2 \tag{2.2}$$

where α_{sx} and α_{sy} are moment coefficient of slab obtained from Table 3.14 of Part 1 of BS8110 (1985); n is the ultimate expressed in load per unit area defined as $n = 1.4\ G_k + 1.6\ Q_k$; and L_x and L_y are length of the edges in action in which it implies to shorter and longer span respectively.

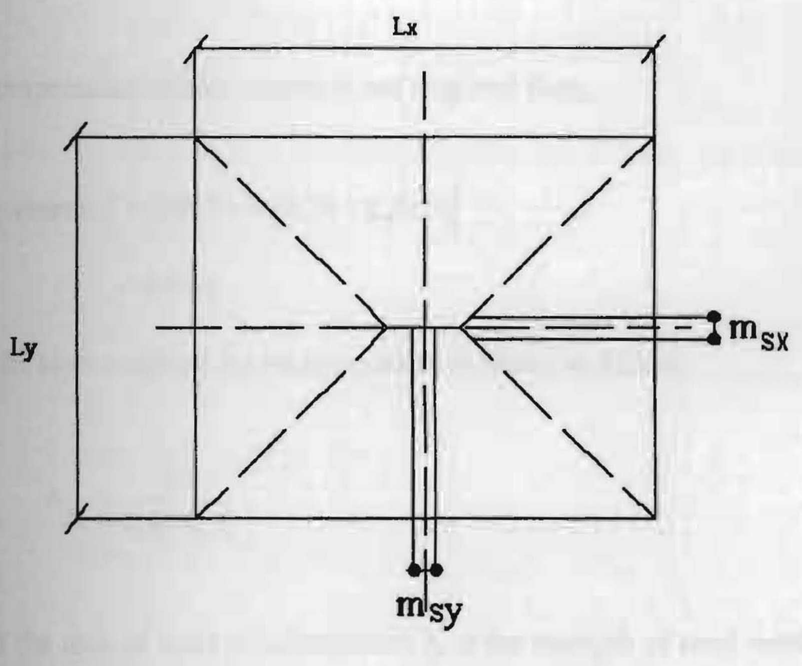

Fig. 2.2 Location of maximum moment.

Referring to Fig 2.2, tension reinforcement can be designed based on Clause 3.4.4.4 of Part 1 of BS8110 (1985). The design of tension reinforcement of a slab is similar to design formulae for a rectangular beam. The following equations are based on simplified stress diagram in Figure 3.3 of Part 1 of BS8110 (1985).

Take K' as a reference for design constant in which it is expressed as;

$K' = 0.156$ if redistribution does not exceed 10% or

$K' = 0.402\ (\beta_b - 0.4) - 0.18(\beta_b - 0.4)^2$ if redistribution exceeds 10%

The equation is given as;

$$K = \frac{M}{f_{cu}\, b d^2} \qquad (2.3)$$

where K is the design constant, M is the ultimate moment per unit area, b is the width of the slab, d is the effective depth, f_{cu} is the concrete strength expressed in Newton per unit area (N / mm^2).

If $K < K'$, compression reinforcement is not required then,

$$\text{Leverarm}, Z = d\left[0.5 + v\left(0.25 - K/0.9\right)\right]$$

$$< 0.95d$$

Therefore the steel required for reinforcement is stated as follow;

$$A_s = \frac{M}{0.87 f_y Z} \qquad (2.4)$$

where A_s is the area of steel reinforcement; f_y is the strength of steel reinforcement in N / mm^2; Z is the lever arm.

If K > K', compression reinforcement doubly reinforcement is required,

Similarly,

$$\text{Leverarm}, Z = d\left[0.5 + v\left(0.25 - K/0.9\right)\right]$$

$$< 0.95d$$

Therefore the total required steel reinforcement area is defined as;

$$A_s' = \frac{(K - K')\, f_{cu}\, b\, d^2}{0.87\, f_y\, (d - d')} \tag{2.5}$$

where d is the effective depth for tension reinforcement and d' is the effective depth for compression reinforcement.

Then total steel area if compression reinforcement is required is;

$$A_s = \frac{M}{0.87\, f_y\, Z} + A_s' \tag{2.6}$$

2.2.3 Shear Reinforcement

When shear stresses based on a normal loads is not critical, shear reinforcement for resisting shearing force is not required. The application of shear reinforcement is done when a thick slab is heavily loaded and the thickness exceeds 200mm. Checking for shear reinforcement is based on Clause 3.5.5 of Part 1 of BS8110(1985). Nominal design shear stress is defined as;

$$v = \frac{V}{b\, d} \tag{2.7}$$

Where V is the shear force due to design ultimate load or concentrated load.